[12-05-14B-T10]

Linear functions - Composition

The set of integers is a commutative group under the operation of addition, because the set is closed under the operation of addition and addition on the integers is commutative, associative, there exists an identity element, 0, and for every integer a there exists an inverse element $-a$.

The set of integers is *not* a group under the operation of multiplication, because there are integers for which no inverse element exists. For example, there is no integer a such that $3 \times a = 1$.

The table below defines an operation, *, on the set $\{a, b, c\}$. Verify that the set together with the operation * is a commutative group.

*	a	b	c
a	a	b	c
b	b	c	a
c	c	a	b

The groups with which you are familiar are commutative (also called abelian) groups, there are many important mathematical structures that fail to be a commutative group because the group operation is not generally commutative on the set. Such systems are called non-commutative (non-abelian) groups.

QUESTION. Is the set of linear functions, i.e. all functions of the form $\boldsymbol{f(x) = a\,x + b}$, a commutative or even a non-commutative, group under the operation of composition, $\boldsymbol{f \circ g}$. As per the custom, please prove your answer.